

## A Robust Kalman Filter Based Sensorless Vector Control of PMSM with LMS Fuzzy Technique

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**Abstract**—In this paper a fuzzy logic technique optimized with Least mean square (LMS) algorithm is used for the vector control of Permanent magnet synchronous motor (PMSM). It is made sensorless by the use of Extended Kalman filter (EKF) algorithm which estimates the speed, rotor position, direct and quadrature axis current of PMSM. Unlike in the previous approaches the present work uses only the sensed line currents as measurements, and thus following a blind system identification approach. The Least mean square algorithm incorporated along with fuzzy inference, optimizes the weights used for combining the rules, which in turn makes the controller more efficient. The results show the improvement in control algorithm when LMS technique is incorporated with fuzzy decision process. The Lyapunov exponent calculated on the phase plane generated by direct axis current, quadrature axis current and motor speed, shows that the system is always asymptotically stable.

**Key words** — blind system identification, Extended Kalman filter, Fuzzy logic speed control, least Mean Square algorithm, Lyapunov exponent.

### 1. Introduction

High torque to inertia ratio, superior power density, high efficiency and many other advantages made PMSM the most widely acceptable electrical motor in industrial applications. The invention of Vector control made the ac drives equivalent to DC drives in the independent control of flux and torque. To facilitate vector control the stator quantities are resolved into components which rotate in synchronism with the rotor. For this transformation of stator quantities into synchronously rotating frame, the accurate knowledge of speed and rotor position is required. It usually requires mechanical sensors for measurement of speed in variable speed applications. But these types of shaft mounted mechanical sensors will make the system more complex and moreover reduces the reliability of the drive system. Accordingly, sensor less operation of PMSM has been receiving wide attention recently [1] in variable speed drives.

In this paper, we propose to estimate the motor quantities like speed, flux vector position, currents in direct and quadrature axes, from the measurements of three phase stator currents using an Extended Kalman filter (EKF). The proposed EKF estimation does not

require the information of the input voltage at all and it makes it different from similar other works[2]. So the present estimation treats the whole problem as blind system identification. The main problem associated with the EKF is its dependence on the parameters like the initial state, initial state co- variance, the measurement noise and the plant noise. The convergence is highly dependent on the choice of the covariance matrices that appear in EKF algorithm. Here the measurements to the EKF are the output of space vector PWM inverter, which is non sinusoidal and rich in harmonics, there by degrading the performance of EKF. To overcome this problem, in this paper a Bayesian approach is used in modifying the elements of the measurement covariance matrix, on the run [4]. The results show the robustness of the resulting implementation of the EKF.

Fuzzy control is different from the traditional PI control in the sense that it does not depend on precise system mathematical model [4]. The fuzzy logic technique has become very popular in the control of ac drives because of the flexibility in accommodating overlapping information in the definition of terms. Standard algorithm computes a fuzzy function on the basis of the error and change in error of the set speed and the estimated speed using a set of rules. The usual approach is to compute a fuzzy function on the error between the set speed and the estimated speed using a set of rules. In the usual approach all the rules are fired with equal weightings. As an optimization to FLC following [6], in this paper a gradient descent method is used, to adjust the weighting of each of the rules of the fuzzy controller, which minimizes the square error between the rotor speed and the reference speed. Least mean square based adaptive fuzzy along with Robust EKF is demonstrated in an environment, where the PMSM runs in Simulink and rest of the speed estimation and the control run concurrently in Matlab. And the results are compared with conventional Fuzzy logic used with conventional Kalman filter to show the improvement in performance. The stability of the controller is confirmed by calculating the Lyapunov exponent value.

### 2. Mathematical model of PMSM

The dynamic model developed on a synchronously rotating reference frame describes better the behavior of the motor for the vector control. Therefore the stator variables are transformed into a synchronously rotating d-q frame. The stator of the PMSM is similar to that of the wound rotor synchronous motor. The back emf produced by a permanent magnet is similar to that produced by an excited coil. A PMSM can be mathematically represented by the following equation in the d-q axis synchronously rotating rotor reference frame for assumed sinusoidal stator excitation [7]:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} R + pL_q & \\ -p\omega_r & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \quad (1)$$

$$d\theta_e/dt = P \omega_r \quad (2)$$

$$T_e = 3P/2 [ i_d i_q ] \quad (3)$$

$$T_e = p \omega_r + T_l \quad (4)$$

Where  $v_d$  and  $v_q$  are the d, q axis voltages,  $L_d$  and  $L_q$  are the d,q axis inductances and  $i_d$  and  $i_q$  are the d,q axis stator currents, respectively. The other parameters are:

- $R$  : the stator resistance per phase
- $\lambda_m$  : the constant flux linkage due to rotor permanent magnet,
- $\omega_r$  : the angular rotor speed,
- $\theta_e$  : the rotor position in electrical degrees,
- $P$  : the number of pole pairs of the motor,
- $p$  : the differential operator,
- $T_e$  : the developed electric torque,
- $T_l$  : the load torque,
- $\sigma$  : the rotor damping coefficient,
- $J$  : the inertia constant

The current control is made possible through a vector control approach. In order to make the PMSM system linear, the d axis current is set to zero. So control of PMSM will become as easy as that of a DC motor. The d-q axis currents are related to the three phase stator currents by the equation:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\sin \theta_e & \cos \theta_e \\ \cos \theta_e & \sin \theta_e \end{bmatrix} \begin{bmatrix} 0 & 1/\sqrt{3} & -1/\sqrt{3} \\ 2/3 & -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (5)$$

### 3. Estimation of speed and rotor position using robust extended Kalman filter

The Extended Kalman filter [8] is an optimal recursive algorithm suitable to estimate the state of nonlinear dynamic systems. The system is described by the following state equations:

$$\dot{x} = f(x) + Bu_k \quad (6)$$

$$y = Cx + v_k \quad (7)$$

where  $v_k$  and  $w_k$  are the zero mean white Gaussian noise. And the state vector,  $x = [i_d \ i_q \ \omega_r \ \theta_e]$ . The measurements are the three phase stator currents  $[i_a \ i_b \ i_c]$ . And  $u$  is the input voltages  $v_d$  and  $v_q$ . Here the state vector  $x$  is augmented with the voltage inputs  $v_d, v_q$ . Then the new state vector  $X$  becomes  $[i_d \ i_q \ \omega_e \ \theta_e \ v_d \ v_q]$ . In discrete form the augmented state model is represented as:

$$\begin{bmatrix} i_d(k) \\ i_q(k) \\ \omega_r(k) \\ \theta_e(k) \\ v_d(k) \\ v_q(k) \end{bmatrix} = \begin{bmatrix} 1 - \frac{R}{L T_s} & \omega_r T_s & 0 & 0 & T_s/L & 0 \\ -\omega_r T_s & 1 - \frac{R}{L T_s} & (-\phi_f/L) T_s & 0 & 0 & T_s/L \\ 0 & (\frac{3\phi_f}{2J}) P^2 T_s & (-\frac{B}{J}) T_s & 0 & 0 & 0 \\ 0 & 0 & T_s & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d(k-1) \\ i_q(k-1) \\ \omega_r(k-1) \\ \theta_e(k-1) \\ v_d(k-1) \\ v_q(k-1) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \\ w_3(k) \\ w_4(k) \\ w_5(k) \\ w_6(k) \end{bmatrix} \quad (8)$$

The mathematical model of PMSM is mutually coupled and hence nonlinear [7]. In the present work, an Extended Kalman filter is used for estimating the speed and the rotor position, from the non linear system given in (8), based on the measured values of line currents. For a given sampling time  $T_s$ , both the state estimate  $\hat{x}_{k/k}$  and its covariance matrix  $P_{k/k}$  are generated by the filter through a two step loop predictor corrector process.

The corrector algorithm starts with an initial value of  $x_0$  and follows as below

1. Computation of the Kalman gain  $K_k = (P_{k/k} H_k^T (H_k P_{k/k} H_k^T + R)^{-1})$  where  $K_k$  is the Kalman gain for  $k^{\text{th}}$  iteration and  $R = X =$  and  $R$  is a constant measurement noise covariance matrix.
2. Update estimate with the measurements  $\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k (y_k - H_k \hat{x}_{k/k-1})$

3. Updating the error covariance as  $P_{k/k} = (I - H_k K_k) P_{k/k-1}$ .

where  $I$  is a identity matrix.

The predictor algorithm involves

1. Projecting the state error covariance matrix ahead for the next iteration as  $P_{k/k+1} = (I - H_k K_k) P_{k/k} + Q$ .

where  $X = \begin{bmatrix} \omega \\ i_q \end{bmatrix}$  and  $Q$  is a constant process covariance matrix.

2. And the state is projected ahead as

$$X_{k+1} = f(X_k)$$

Beginning the iteration with an initial value of  $x$  is  $X(0)$  and the Covariance  $P(0)$  the Kalman filter estimates the values of flux angle  $\theta$  and the speed  $\omega$ . The convergence of Kalman filter is highly effected by the choice of  $X(0)$ ,  $Q$  and  $R$ . Usually these matrices are chosen by trial and error approach. Beginning the iteration with an initial value of  $x$  is  $X(0)$  and the Covariance  $P(0)$  the Kalman filter estimates the values of flux angle  $\theta$  and the speed  $\omega$ . Since the measurements contain a large number of harmonics the performance of the EKF is often tainted. In this paper, the EKF is made more robust using an alternate method, based on Bayesian approach [4]. A scalar weight  $G_k$  is introduced for each data sample such that the variance of  $R$  is weighted with  $G_k$ , given by:

$$G_k = \frac{a}{b + (Z_k - \hat{h}_k \hat{x}_k/k)}$$
 (9)

$a$  and  $b$  are taken as constants equal to one. This weight  $G_k$  is utilized to scale down the measurement covariance matrix  $R$ , before it goes to EKF algorithm for the calculation of Kalman gain i.e.  $R$  is modified as  $R/G_k$ .

During simulations, it has been observed that the robust EKF eliminates the need for manual parameter tuning of measurement covariance matrix in EKF equation. The results also show that the Kalman filter has become robust against input variation, with this technique.

#### 4. LMS fuzzy logic speed controller

The over control of the speed is realized in terms of  $e$  and  $\Delta e$ . The error in speed and the rate of change of

speed error are considered as the input linguistic variables and the quadrature axis current is considered as the output linguistic variable. The support for all the fuzzy variables viz.  $e$  and  $\Delta e$ , variable of output ie  $i_q$  is scaled to  $[-1, 1]$ . In this case 7 membership functions are used, viz., NB, NM, NS, ZO, PS, PM, PB as shown in Fig.1

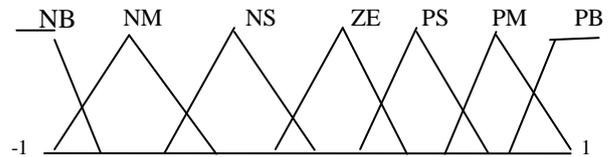


Figure. 1 Membership function for computing quadrature axis current.

Rules for computing the quadrature axis current is given below

TABLE 1  
Rules of quadrature axis current

| $i_q$      | e  |    |    |    |    |    |    |
|------------|----|----|----|----|----|----|----|
| $\Delta e$ | NB | NM | NS | ZO | PS | PM | PB |
| NB         | NB | NB | NB | NB | NM | NS | ZO |
| NM         | NB | NB | NB | NM | NS | ZO | PS |
| NS         | NB | NB | NM | NS | ZO | PS | PB |
| ZO         | NB | NM | NS | ZO | PS | PM | PB |
| PS         | NM | NS | ZO | PS | PM | PB | PB |
| PM         | NS | ZO | PS | PM | PB | PB | PB |
| PB         | ZO | PS | PM | PB | PB | PB | PB |

The block diagram representation of the controller is shown in Fig 2.

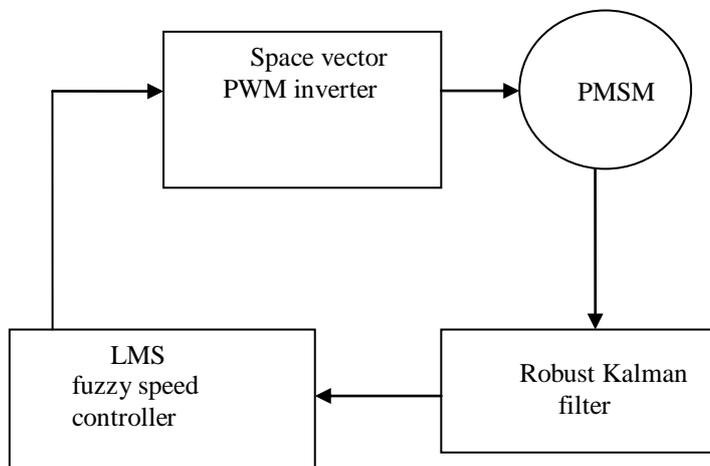


Figure 2 Block diagram representation of the control system

Using the product inference rule and central average defuzzifier method the fuzzy output is expressed as:

$$q = \frac{\sum_{i=1}^n \mu^i C_m}{\sum_{i=1}^n \mu^i} \quad (10)$$

Where  $n$  is the no. of rules,  $\mu^i$  is the weighting factor which is to be adjusted and  $C_m$  is the membership function of  $i^{\text{th}}$  rule. The weighting parameters  $C$  of each rule are adjusted in order to minimize the square of the error between the instantaneous rotor speed and reference speed. For this the gradient descent method is used. The instantaneous error function is defined as:

$$J(k+1) = \frac{1}{2} [\omega_m(k+1)]^2 \quad (11)$$

where  $\omega_m$  is the reference motor speed and the parameters of  $C_j$  are adjusted with [6]:

$$\Delta C_j(k) \approx \alpha \text{Sgn}(B) \quad (12)$$

where  $\alpha$  is the learning rate.  $B$  is the motor parameter,

$$B = \dots ; \text{ and}$$

$$A = \exp(\dots)$$

where  $T$  is the sampling period.  $K_i$  and  $K_u$  are the scaling factors. The current command  $i_q^*$  is obtained by the output of AFC as:

$$i_q^*(k) = i_q^*(k-1) + k_u q(k) \quad (13)$$

### 5. Computation of the Lyapunov exponent

The Lyapunov exponent is an important indicator of the stability of a nonlinear system. Given a continuous dynamical system in an  $n$  dimensional phase space, we monitor the long term evolution of an infinitesimal sphere of initial condition. The  $i$ th one dimensional Lyapunov exponent is then defined as follows:

$$\lambda_i = \dots \quad (14)$$

The signs of the Lyapunov exponents provide a qualitative picture of a system's dynamics.

- If  $\lambda_i = 0$ ; a marginally stable orbit.
- If  $\lambda_i < 0$ ; a periodic orbit or a fixed point
- If  $\lambda_i > 0$ ; Chaos

### 6. Simulation and Results

Simulation of the given PMSM has been carried out using Simulink. The three phase stator currents are taken from the motor model, and given to the robust extended Kalman filter. The Robust EKF estimates the instantaneous motor speed, rotor position, quadrature and direct axis currents and voltages. The estimated motor speed is compared with the reference speed and the error produced and change in the error is given to the Adaptive fuzzy speed controller. Where, it undergoes

the fuzzy inference process according to the rule generated in TABLE1. It gives quadrature axis current as the output linguistic variable. Fuzzy speed controller output is compared with the estimated quadrature axis current and the error produced is passed through a PI current to voltage converter to produce quadrature axis voltage. The direct axis current reference is set to zero and this value is compared with the estimated direct axis current and the error produced is passed through another PI current to voltage converter which produces direct axis voltage. The direct and quadrature axis voltages are converted into two axis stator voltages using the inverse park transformation. The two axis stator voltages are utilized to trigger the space vector PWM inverter. The space vector PWM inverter drives the PMSM. The approach towards the estimation of speed and rotor position is stable and converges very fast. It needs only the three phase stator current values which are easily available from the machine.

In order to check the stability of the system Lyapunov exponent was calculated and found that the system is always asymptotically stable.

The Lyapunov exponent values for the three phase vectors direct axis current, quadrature axis current, motor speed are as follows: -8.2999, -6.9554, -5.2624.

The phase plane trajectory of these three vectors is plotted in Figure 7.

The Robust EKF algorithm is developed in Matlab and integrated with Simulink using embedded Matlab facility. The parameters used for simulation are discussed below. The PMSM parameter used in this paper is **1.1 KW, 4 poles,  $R=2.875\Omega$ ,  $L=0.423H$ ,  $\Phi=1.7\text{wb/m}^2$ ,  $J=0.008\text{Kg-m}^2$** . During the simulation, it was seen that convergence for the speed estimating Kalman filters is highly dependent on the initial values viz.  $X(0)$  and  $P(0^-)$ . The values of those matrices are given as

$$X(0) = [0; 0; 0; 0; 0; 0], R = \text{dia} [10; 10; 10]$$

$$P(0^-) = \text{dia} [200; 200; 200; 200; 200; 200]$$

The sampling time chosen is  $4.5e^{-5}$  S. The values of proportional and integral gain constants used are,  $K_p=60$ ,  $K_i=2$ ; for inner quadrature axis current controller.  $K_p=40$ ,  $K_i=2$ ; for inner direct axis current controller.

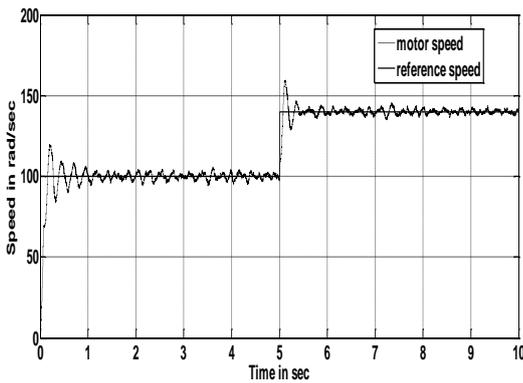


Figure 3. Step response of speed with LMS estimation of  $C_m$  in the fuzzy control strategy

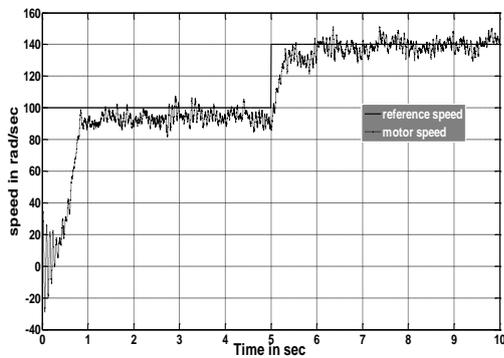


Figure 4. Step speed response with fuzzy control, without LMS estimation of  $C_m$

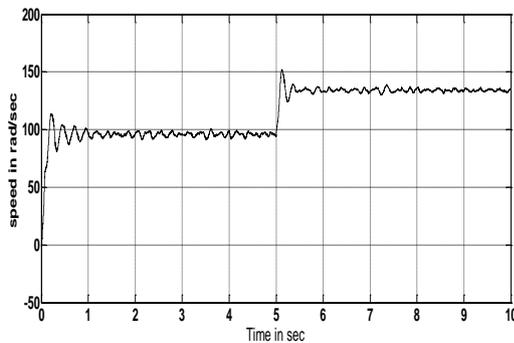


Figure 5. Estimated speed with Robust EKF

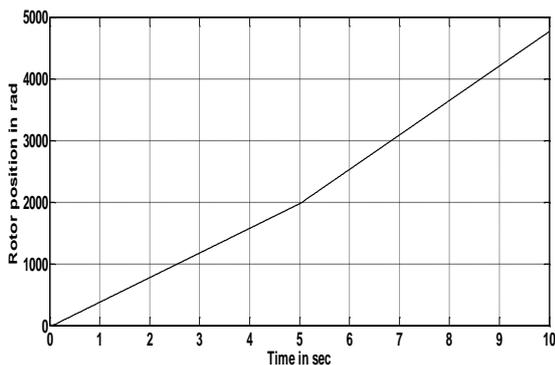


Figure6. Estimated rotor position with Robust EKF

Fig 3 shows the step response of mechanical speed of PMSM with a LMS Fuzzy, Robust EKF, from the result it is clear that the speed converges within 0.5 sec time with an error of 1%.hence it is very quick in operation compared to the conventional fuzzy technique. Fig 4 shows the same step response with conventional fuzzy, there it takes nearly 1 sec time to converge. Fig 5 shows the estimated motor speed for the step speed change of 100 rad/sec to 140 rad/sec with Robust EKF. From the results it is clear that the LMS technique incorporated along with Fuzzy logic made it quicker compared to the conventional case. In the speed graph it is clear that the estimated speed is completely coincident with the running speed of motor. Figure 6 shows that the slope of the estimated rotor position also changes when the speed steps into a new value. These step changes are required in electric vehicle application of the PMSM. The information regarding rotor position is utilized in Park inverse transformation. Fig 7 shows that the system is asymptotically stable.

The Robust EKF completely eliminates the difficulties in convergence of EKF algorithm. One of the main distinction from similar other works in literature is that the proposed method uses only three stator current measurements of PMSM in order to estimate six quantities in the state vector  $X$ , at a time. In order to assess the stability of the proposed control strategy, the phase plane corresponding to  $i_d, i_q$  and  $\omega$ . Fig. 7 illustrates the phase plane. The negative values of the Lyapunov exponents viz.  $-8.2999, -6.9554, -5.2624$ , confirm the stability.

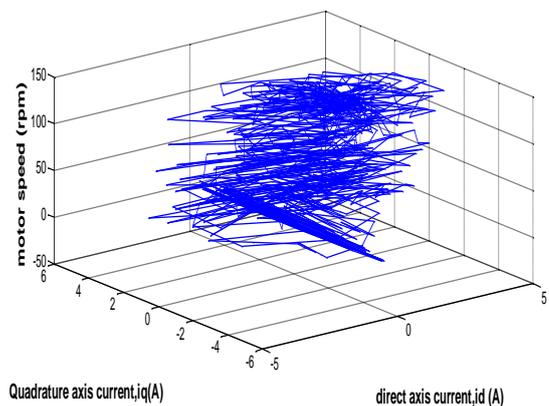


Figure 7 Phase plane trajectory

## 7. Conclusions

A Robust EKF estimator along with Fuzzy logic speed controller with an LMS estimate of the weighting coefficients has been successfully implemented in SIMULINK/MATLAB environment. The performance shows that the the change in speed is achieved within 0.5 S, with total stability in the control. The results illustrate the efficacy of the Bayesian approach for the elimination of outlier problem in the

EKF algorithm. The gradient descent algorithm used in the optimization of the combining of fuzzy rules included in the controller helps to catch up the sudden changes in speed very quickly within less than 500ms time. The Kalman filter proves its efficacy as an estimator without the knowledge of inputs, as a blind system identification approach. The controller performance has been tested for various step changes in speed, in the Simulink environment and established fast convergence and the stability at various speeds. .

## 8. Acknowledgement

The first author would like to thank the Institute of Human resource Development, Kerala, India for permitting her to do research work.

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